DTU Library

Simplified Laws of Similarity for Wind Turbine Rotors

Petersen, H.

Publication date:
1984

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
Petersen, H. (1984). *Simplified Laws of Similarity for Wind Turbine Rotors*. Danmarks Tekniske Universitet, Risø Nationallaboratoriet for Bæredygtig Energi.

RISØ-M-2432

SIMPLIFIED LAWS OF SIMIALRITY FOR WIND TURBINE ROTORS

Helge Petersen
The Test Station for Small Windmills

Abstract. Laws of similarity or scaling laws for the characteristics of a wind turbine rotor are of importance to the designer even during the initial design phase of a new wind turbine concept, but are still more so when the designer wants to make adjustments or modifications to a wind turbine, in order to improve the machine, optimize or adapt it to another wind climate than originally anticipated, or eventually redesign it by scaling up or down the rotor size. In the paper some esamples are shown to illustrate the principles, and equations for the laws of similarity are derived under simplified conditions such as neglecting the influence of the variation of the Reynolds Number. Some comments are presented on comparison of stall-and pitch-regulated wind turbines and on two speed operation. The optimization of a stall-regulated wind turbine to different wind climates by variation of the generator size is illustrated by examples.

EDB descriptors: ADJUSTMENTS; DESIGN; DIMENSIONS; OPTIMIZATION; REYNOLDS NUMBER; ROTORS; WIND TURBINES.

UDC 621.548

May 1984
Risø National Laboratory, DK-4000 Roskilde, Denmark

ISBN 87-550-1007-5
ISSN 0418-6435

Risø Repro 1984

CONTENTS Page

NOMENCLATURE

A	Rotor disc area	m^2
C	Scaling factor (Weibull)	$m\ s^{-1}$
C_p	Rotor power coefficient	-
D	Rotor diameter	m
E	Annual energy production	MWh
K	Shape factor (Weibull)	-
N	Rotational speed	RPM
P	Power	kW
R	Rotor radius	m
V	Wind speed	$m\ s^{-1}$
V_{tip}	Rotor tip speed	$m\ s^{-1}$
X	Tip speed ratio	-
λ	Ratio	-
ω	Angular speed	s^{-1}
ρ	Air density	$kg\ m^{-3}$

1. INTRODUCTION

The laws of similarity for the characteristics of a wind turbine rotor are of importance to the designer even during the initial design phase of a new wind turbine concept, but are still more so when the designer wants to make adjustments or modifications to a wind turbine, in order to improve the machine, optimize or adapt it to another wind climate than originally anticipated, or eventually redesign the machine by scaling up or down the rotorsize. In the paper some examples are shown to illustrate the principles, and the equations for the laws of similarity are derived under simplified conditions such as neglecting the influence of the variation of the Reynolds Number.

The examples in the following are all related to one wind turbine rotor named "Rotor 185" referring to the disc area, A= 185 m^2. In this context Rotor 185 is described only by the diameter, D= 15.34 m, and the rotor power coefficient, C_p, shown in Fig. 1. The power coefficient is defined by the equation for the rotor power, P. For convenience the power, P, is in kW for which reason the factor 10^{-3} is introduced in the equation.

$$P = \tfrac{1}{2} \rho V^3 A C_P 10^{-3} \qquad (1)$$

V is the wind speed at hub height, ρ is the air density.

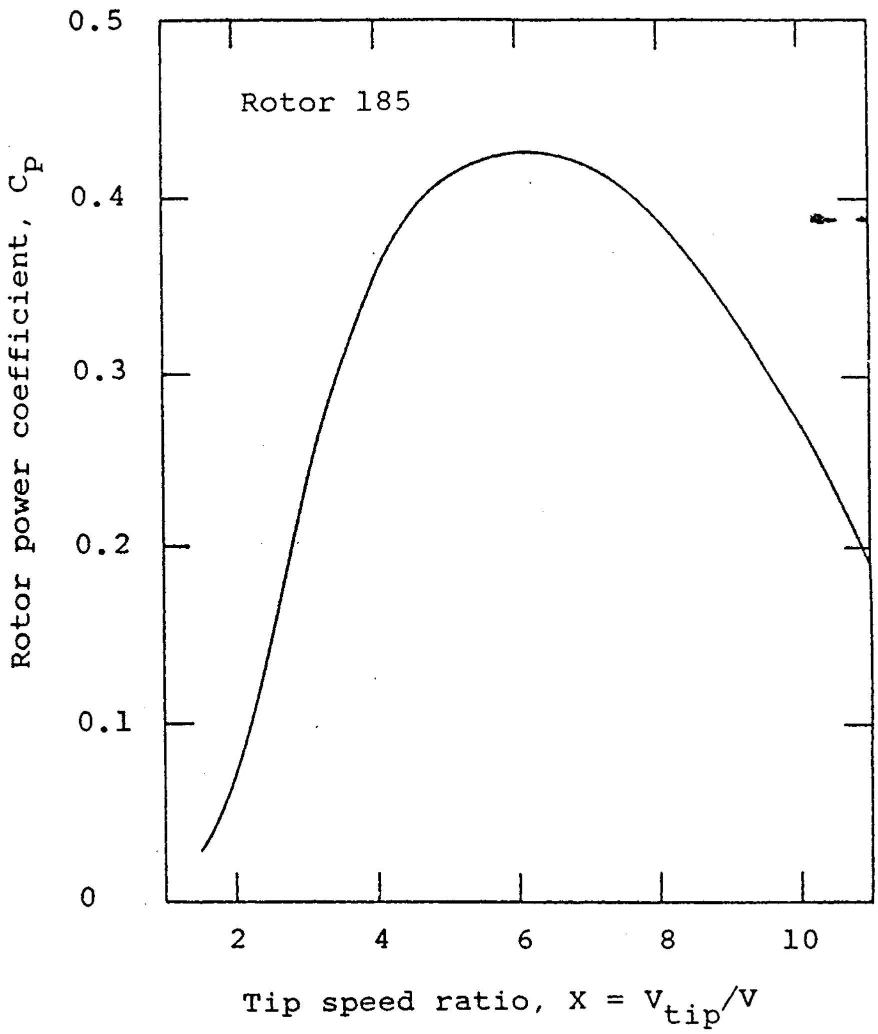
0.5
Rotor 185
0.4
0.3
0.2
0.1
0
Rotor power coefficient, C_p
2
4
6
8
10
Tip speed ratio, $X = V_{tip}/V$

Figure 1.

In Figs. 1 and 2 the rotor power coefficient is shown as a function of the tip speed ratio, X, and the reciprocal, λ, respectively. Defined in this way we have,

$$\lambda = \frac{V}{V_{tip}} = \frac{1}{X} \tag{2}$$

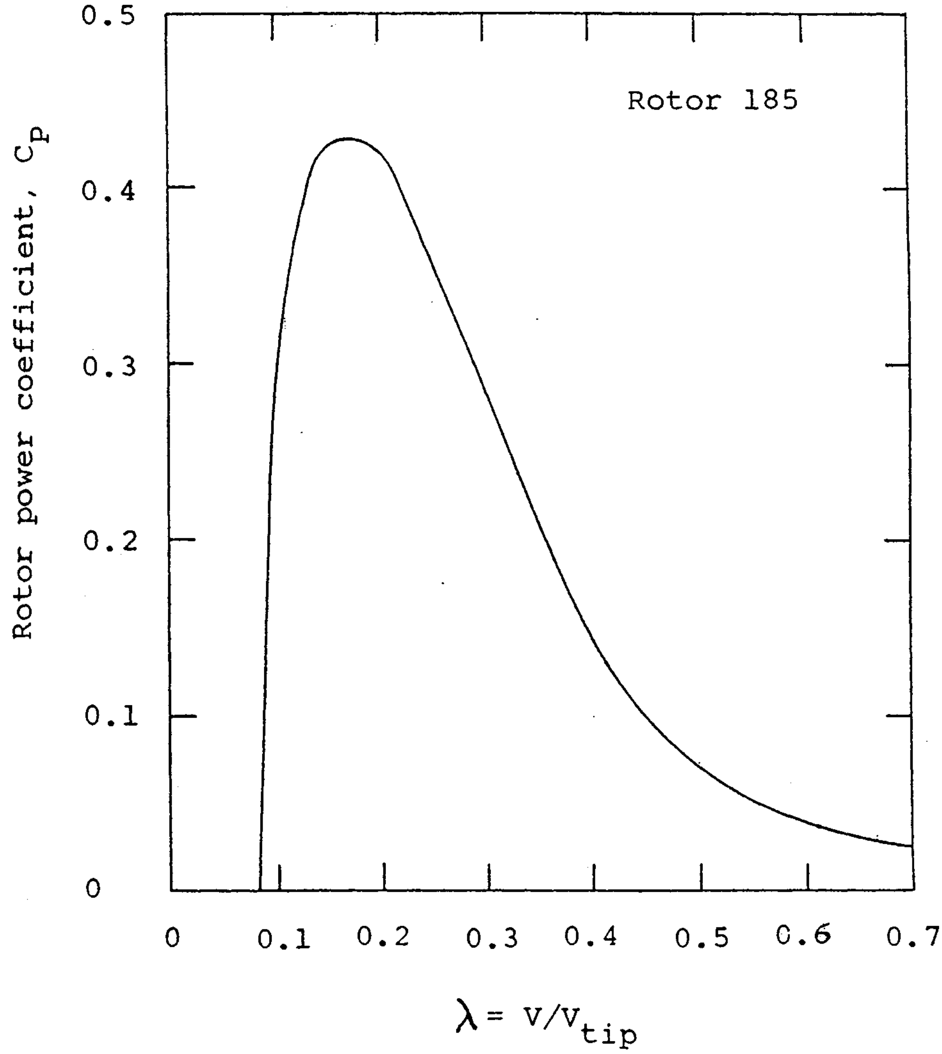

Figure 2.

By multiplying the power coefficient, C_P, by λ^3 another useful dimensionless graph is obtained shown in Fig. 3. The point of maximum (in this case $\lambda^3 C_P = 0.0093$ at $\lambda = 0.4$) represents the condition at which the rotor power is maximum.

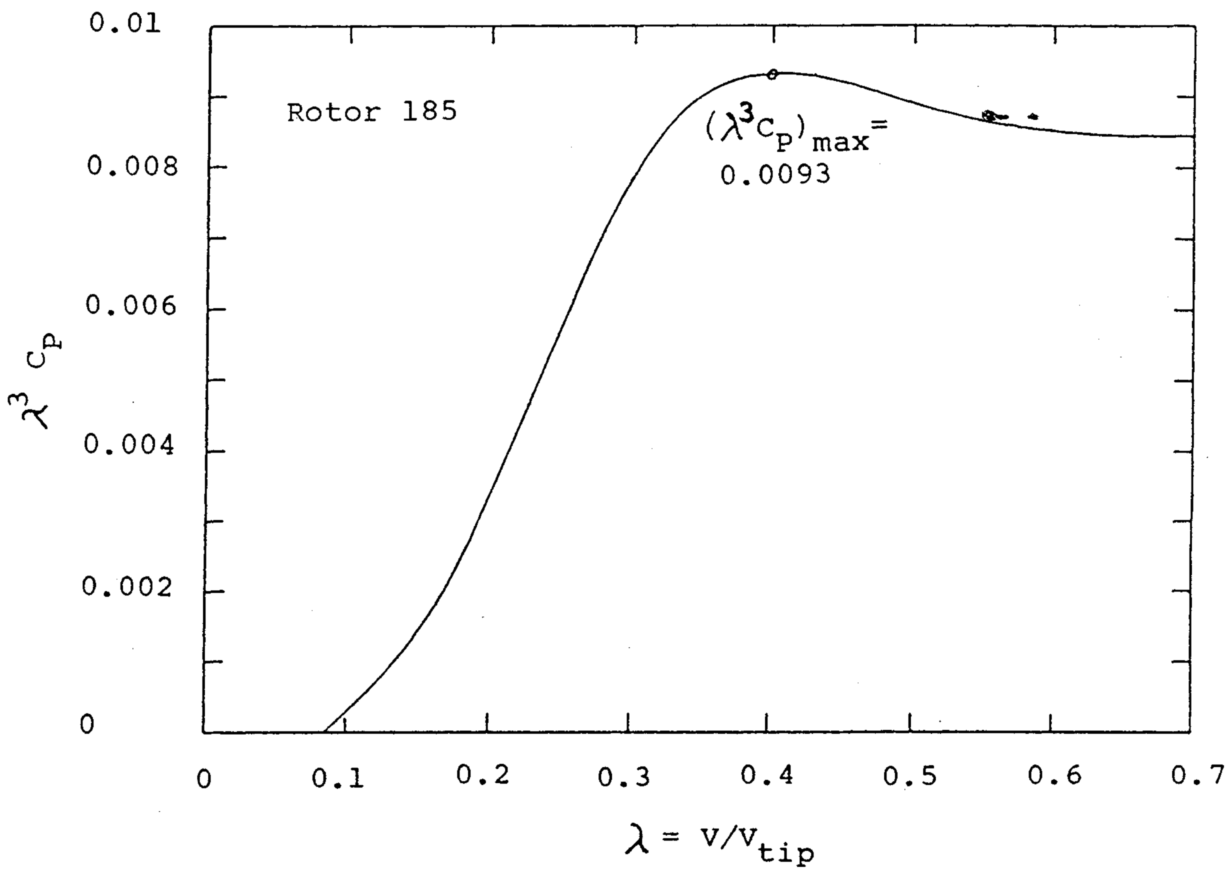

Figure 3.

Equations (3), (4) and (5) show that for a stall-regulated rotor at constant speed the rotor power is in proportion to the term $(\lambda^3 C_P)$ shown in Fig. 3.

$$P = \tfrac{1}{2} \rho A V^3 C_P 10^{-3} = \tfrac{1}{2} \rho A \frac{V^3}{\lambda^3} (\lambda^3 C_P) 10^{-3} \tag{3}$$

$$V = \lambda V_{tip} \tag{4}$$

$$P = \tfrac{1}{2} \rho A V_{tip}^3 (\lambda^3 C_P) 10^{-3} \tag{5}$$

2. VARIATION OF THE ROTATIONAL SPEED.

Figure 2 shows the power coefficient for Rotor 185. Let us now suppose that with sufficient accuracy we could accept that such a relation between λ and C_p holds independently of the rotational speed of the rotor, i.e. that we can neglect the change in Reynolds Number which might have influence on the profile aerodynamic data. Under this condition the derivation of the relations is very simple.

The basic equation (eq.1) can be rewritten:

$$P = \tfrac{1}{2} \rho V^3 \pi R^2 C_P 10^{-3} \tag{6}$$

R is the radius of the rotor.
The angular velocity of the rotor is

$$\omega = 2 \pi N/60 \tag{7}$$

N is the number of revolutions per minute.
The tip speed is

$$V_{tip} = \omega R \tag{8}$$

Equations (1) to (5) leads to equation (11) for the power

$$\lambda = V/V_{tip} = V/(\omega R) \tag{9}$$

$$V = \lambda \omega R \tag{10}$$

$$P = \tfrac{1}{2} \rho (\lambda \omega R)^3 \pi R^2 C_P 10^{-3} = \tfrac{1}{2} \pi \rho \omega^3 R^5 (\lambda^3 C_P) 10^{-3} \tag{11}$$

Therefore, for a given radius, R, and a given set of (λ, C_p) the law of similarity states that,

$$\frac{P_{1,(\lambda,C_p)_1}}{P_{2,(\lambda,C_p)_1}} = \frac{\omega_1^3}{\omega_2^3} = \frac{N_1^3}{N_2^3} \tag{12}$$

If we specifically look at the condition for the maximum power which is a main feature of the stall-controlled rotor, we have (corresponding to the point of maximum of $(\lambda^3 C_p, \lambda)$).

$$\frac{P_{max,1}}{P_{max,2}} = \frac{\omega_1^3}{\omega_2^3} = \frac{N_1^3}{N_2^3} \tag{13}$$

Proceeding with the example, Rotor 185, Fig. 4 shows power curves for some values of the rotational speed, N.

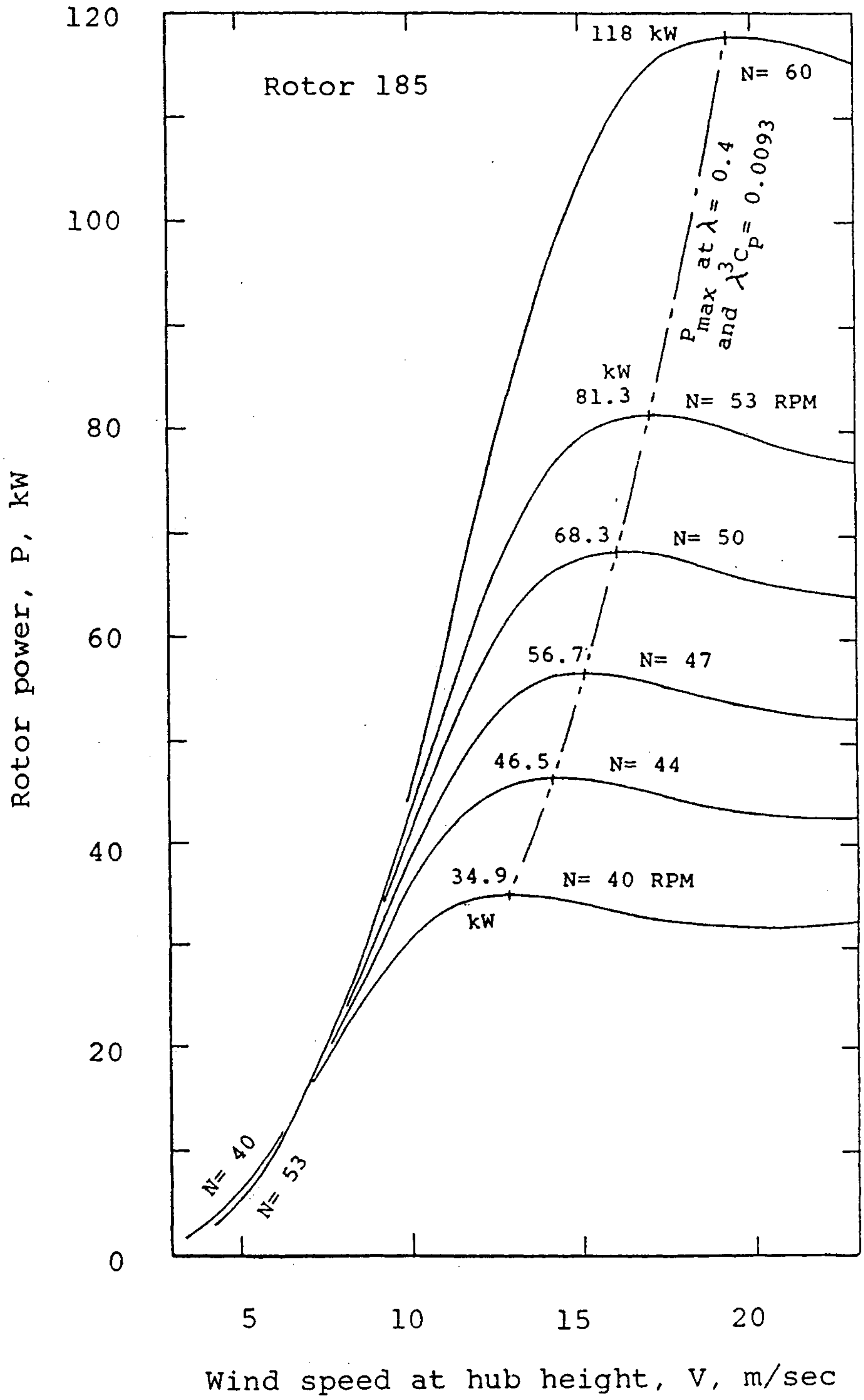

Figure 4.

The figure illustrates the prediction of the power curve: based on the law of similarity.

If the rotor is to be used at another speed of rotation, another generator would be needed, or if inversely another generator size were believed to be optimal, another speed of rotation would be required. According to the equations the rule is simple: The peak power of the rotor varies as the cube of the speed of rotation as shown in Fig. 5 for the example, Rotor 185.

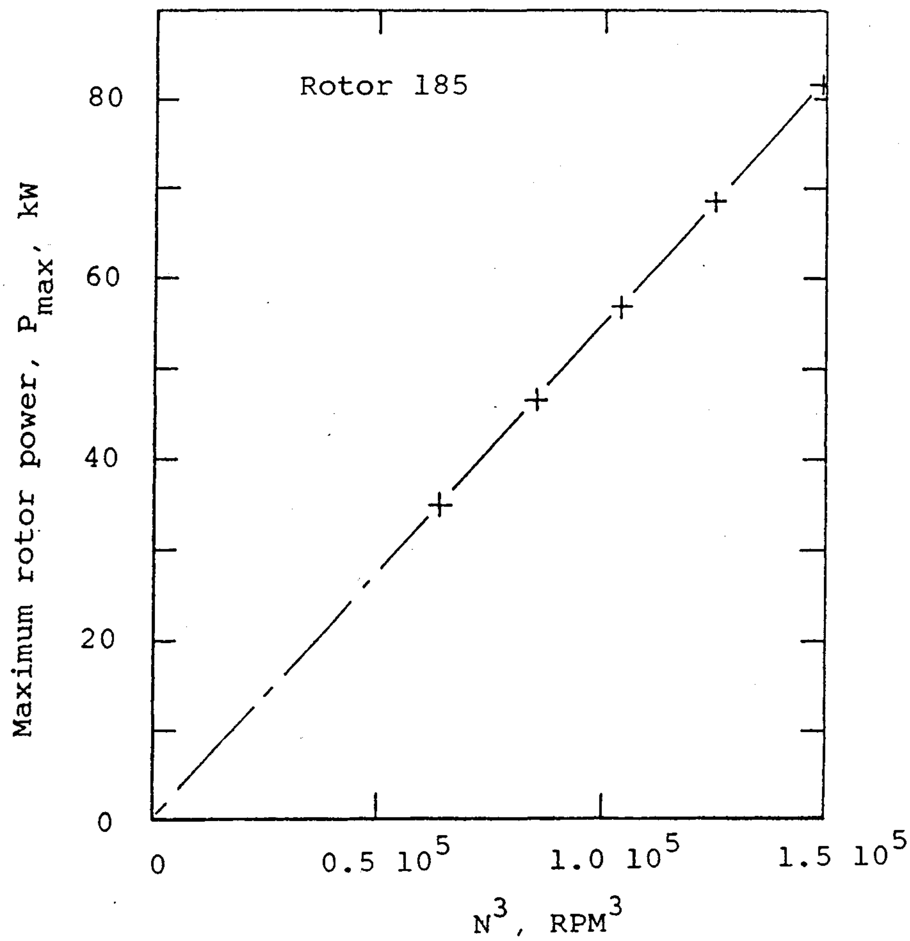

Figure 5.

3. VARIATION OF THE RADIUS OF THE ROTOR.

We can further extend the rule to incorporate a variation of the radius of the rotor.

Let us repeat the conditions of the rule: We deal with a given rotor at a given blade incidence angle relative to the rotor-plane and assume that the variation of the power coefficient versus the tip speed ratio is independent of the rotational speed of the rotor. We now assume further that the C_p curve is valid also for the variation of the radius of the rotor for rotors scaled from the previous one. This again is the assumption that we can neglect the Reynolds Number effect. The equation derived here is strictly correct only if the blades are truly scaled up or down in size in proportion to the change of the radius. However, for small variations of the radius, such as extensions of the blade supports at the hub, this simple scaling law may be sufficiently accurate.

The law of similarity incorporating the variation of the radius of the rotor for any given set of (λ, Cp) is,

$$\frac{P_{1,(\lambda,C_p)_1}}{P_{2,(\lambda,C_p)_1}} = \frac{N_1^3}{N_2^3}\,\frac{R_1^5}{R_2^5} \tag{14}$$

For the maximum power, corresponding to the point of maximum of $(\lambda^3 Cp, \lambda)$ we have

$$\frac{P_{max,1}}{P_{max,2}} = \frac{N_1^3}{N_2^3}\,\frac{R_1^5}{R_2^5} \tag{15}$$

Let us again illustrate the procedure by means of an example and assume that we will scale Rotor 185 up from R = 7.67 m to R = 8.17 m, denoting this Rotor 210 (A = 210 m^2). Figure 3 shows that the maximum of ($\lambda^3 C_p, \lambda$) is $\lambda^3 C_p = 0.0093$, $\lambda \doteq 0.4$. For Rotor 185 at N = 50 RPM we find from equation (11) that P_{max} = 68.2 kW and correspondingly that Rotor 210 at 50 RPM would give P_{max} = 93.5 kW. However, this is a considerably higher power disc loading, 0.445 kW/m^2, compared with 0.369 for Rotor 185. It is more reasonable to keep the disc loading the same and according to equation (15) the rotational speed should then be lowered to 47 RPM, corresponding to a maximum power for Rotor 210 of 77.4 kW. This reduction in rotational speed equals keeping the tip speed constant (at 40.2 m/sec). In other words: Keeping the tip speed constant equals keeping the specific power disc loading constant (all other parameters unchanged). The example is shown below in Fig. 6. The gain then corresponds to the increase of the disc area.

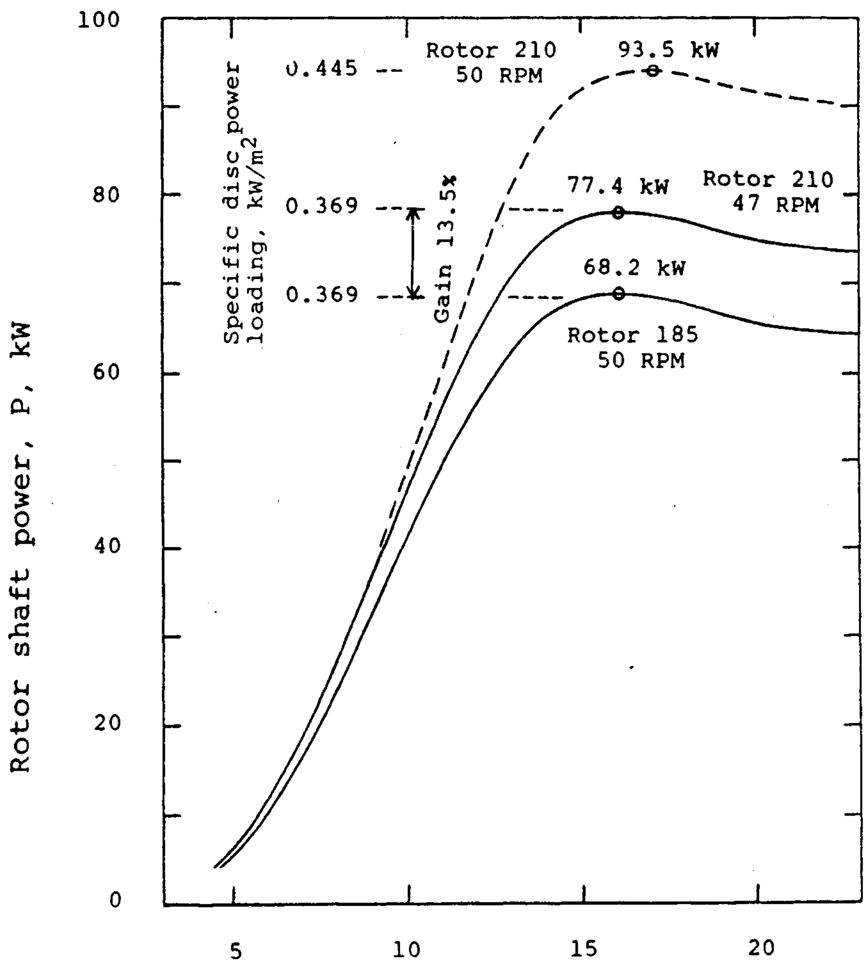

Figure 6.

4. VARIATION OF THE INCIDENCE OF THE BLADES.

Another way to vary the power curve of a stall-regulated rotor is to vary the incidence of the blades (pitch setting) with respect to the rotor plane. The effect of this cannot be generalized as was the case of the influence of the rotational speed or the radius; however, the example shown in Fig. 7 may serve as an indication of the change of the power curve versus the change of the blade setting. The basis of Fig. 7 is the power curve for Rotor 185 at N = 51 RPM. The blade setting for this is denoted 0°, and the figure then gives the power curves when the blade setting is varied from -3° to +3°.

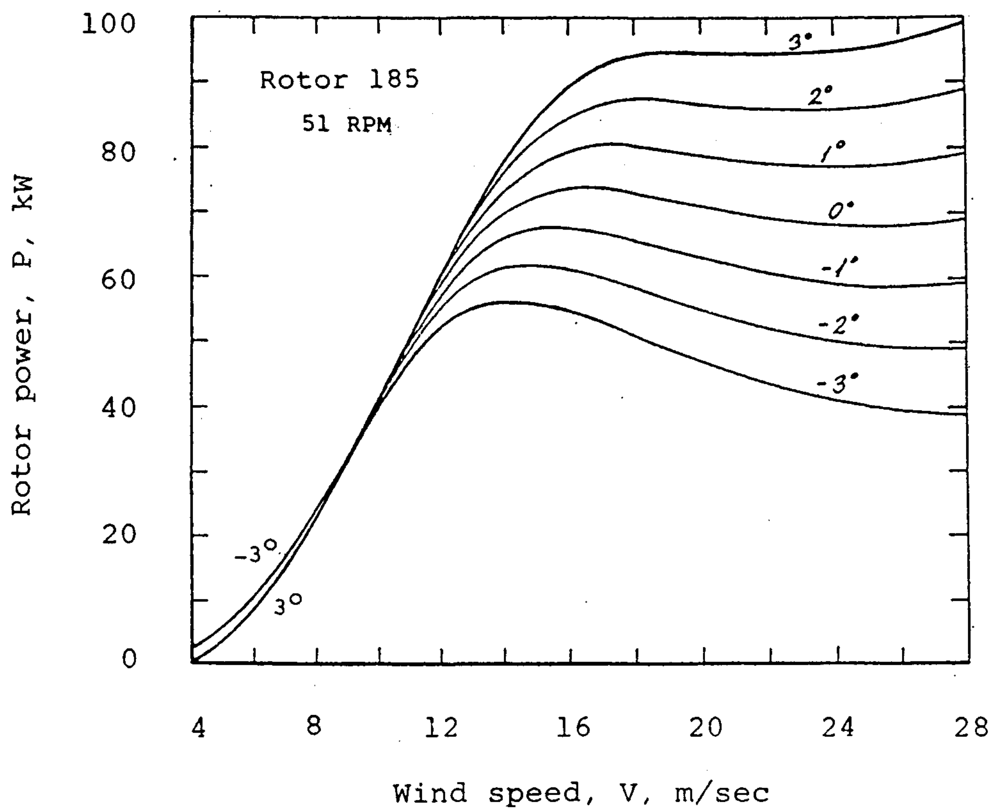

Figure 7.

5. COMMENTS ON COMPARISON OF A STALL- AND PITCH-REGULATED ROTOR AND ON TWO-SPEED OPERATION OF A WIND TURBINE.

Figure 8 shows once more a power curve of the rotor dealt with before, Rotor 185, at N = 51 RPM. The curve is denoted Stall-regulated Rotor. If we imagine that the same rotor should be used for a pitch-regulated machine it would be likely that the speed of the rotor should be increased to 60 RPM (see Fig 4). The corresponding power curve is shown in Fig. 8, denoted Pitch-regulated Rotor.

As can be seen by comparing the two curves in Fig. 8, pitch regulation would increase the efficiency of the rotor at wind speeds of 9 to 15 m/sec. On the other hand it would be at the expense of the more complicated mechanical system and control system.

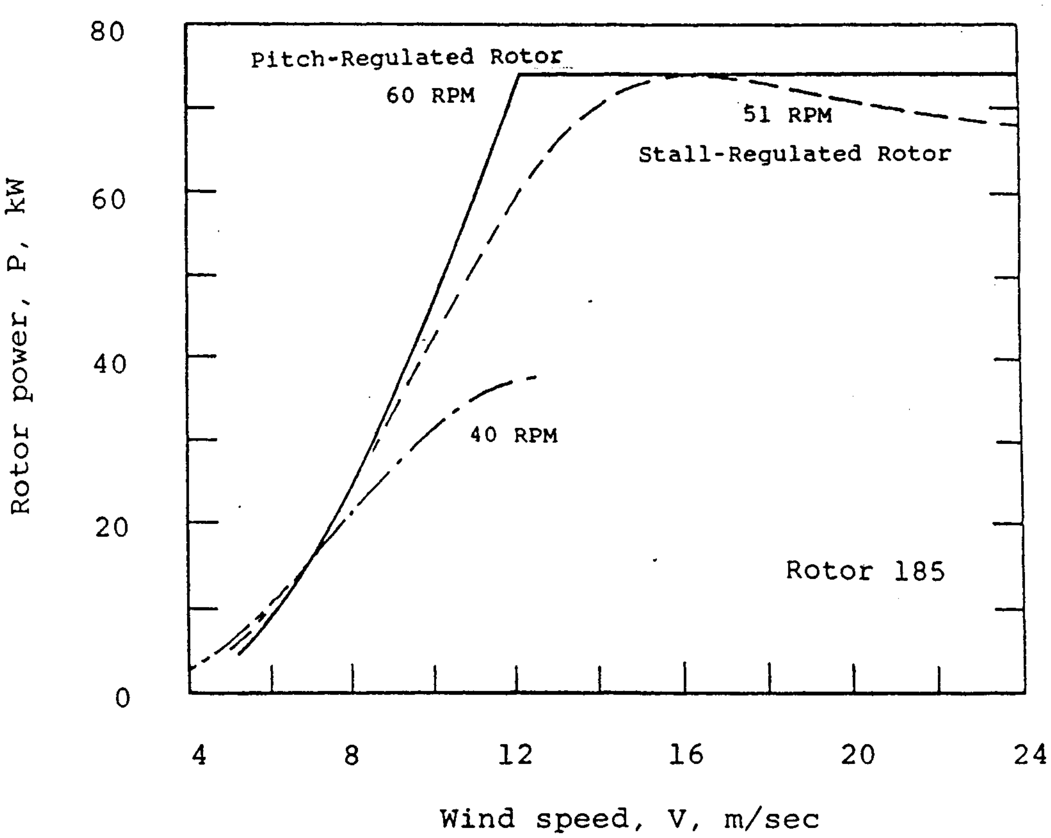

Figure 8.

Another feature to be seen in the figure is the slight, but important, advantage of using a lower RPM at low wind speeds, illustrated by the power curve at 40 RPM. This feature is used in the two-speed operation of the wind turbine, running at the lower RPM at low wind speeds and at the high RPM at high wind speeds. This system can be applied at stall-controlled wind turbines as well as pitch-controlled machines. The aerodynamic advantage may not seem very much and the extra production of electricity resulting from such a two-speed system may seem rather uninteresting. It could, however, gain importance when it is remembered, that the small generator could have lower losses, thus adding further gain, and it could be in a prominent low wind speed regime emphasizing the need to optimize at these low power levels.

6. OPTIMIZATION OF A STALL-REGULATED WIND TURBINE TO DIFFERENT WIND CLIMATES BY VARIATION OF THE GENERATOR SIZE.

In this Section the through going example, Rotor 185, is used to illustrate how important it is for the annual energy production to select the right combination of rotational speed and generator size, adjusted to the wind climate at the expected operating site.

In order to do so, firstly the rotor power curves are converted to power curves for the electrical power output of correspondent complete wind turbines with different generator sizes, and secondly some different wind climates are defined. Then the annual energy production of the wind turbines are calculated.

The curves for electrical power output in Fig. 9 are derived from the rotor power curves in Fig. 4 by assuming a (so far) arbitrarily chosen set of efficiency curves for the gear train and generator. The efficiency curves are described and shown overleaf in Figs. 10, 11 and 12.

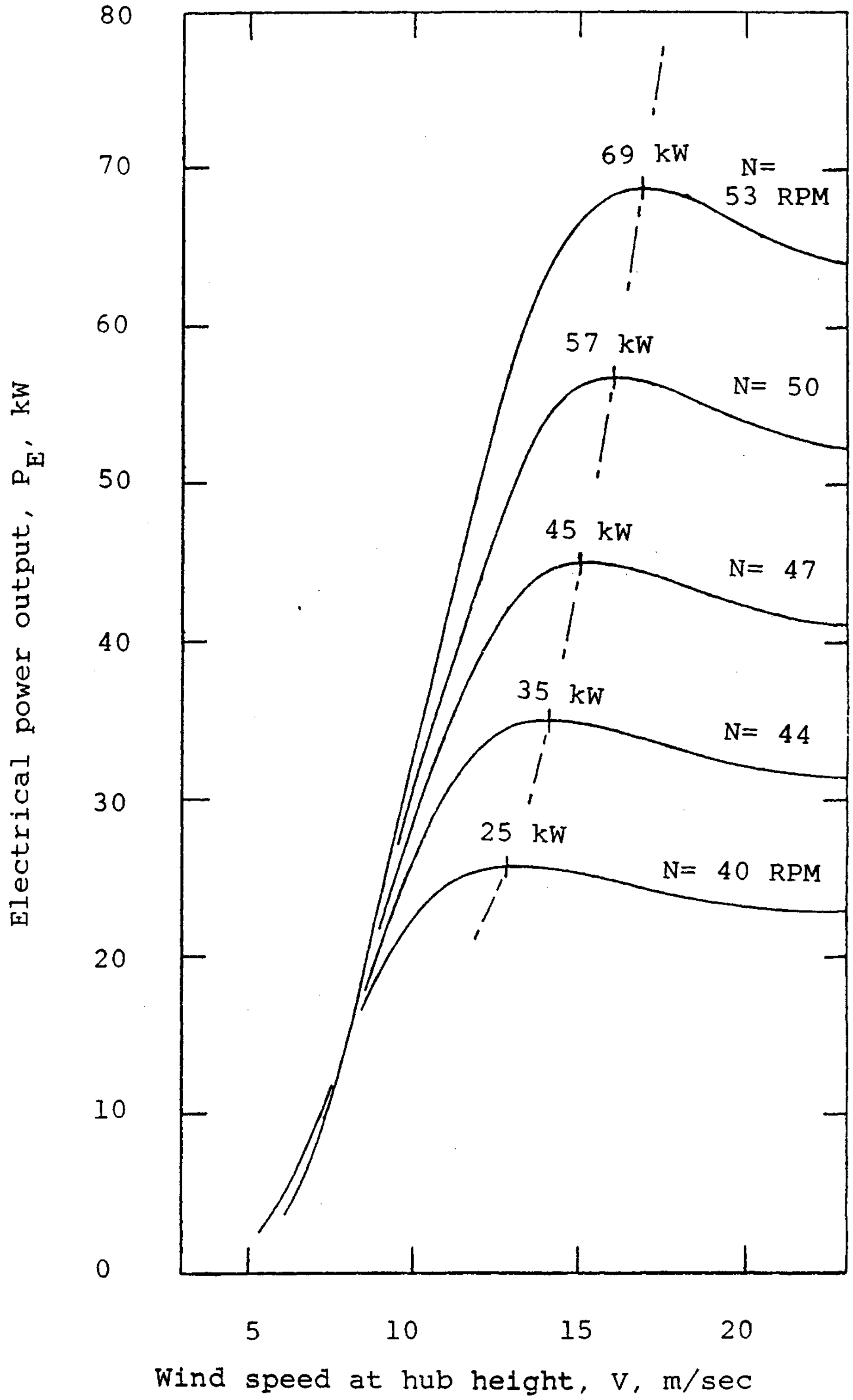

Figure 9.

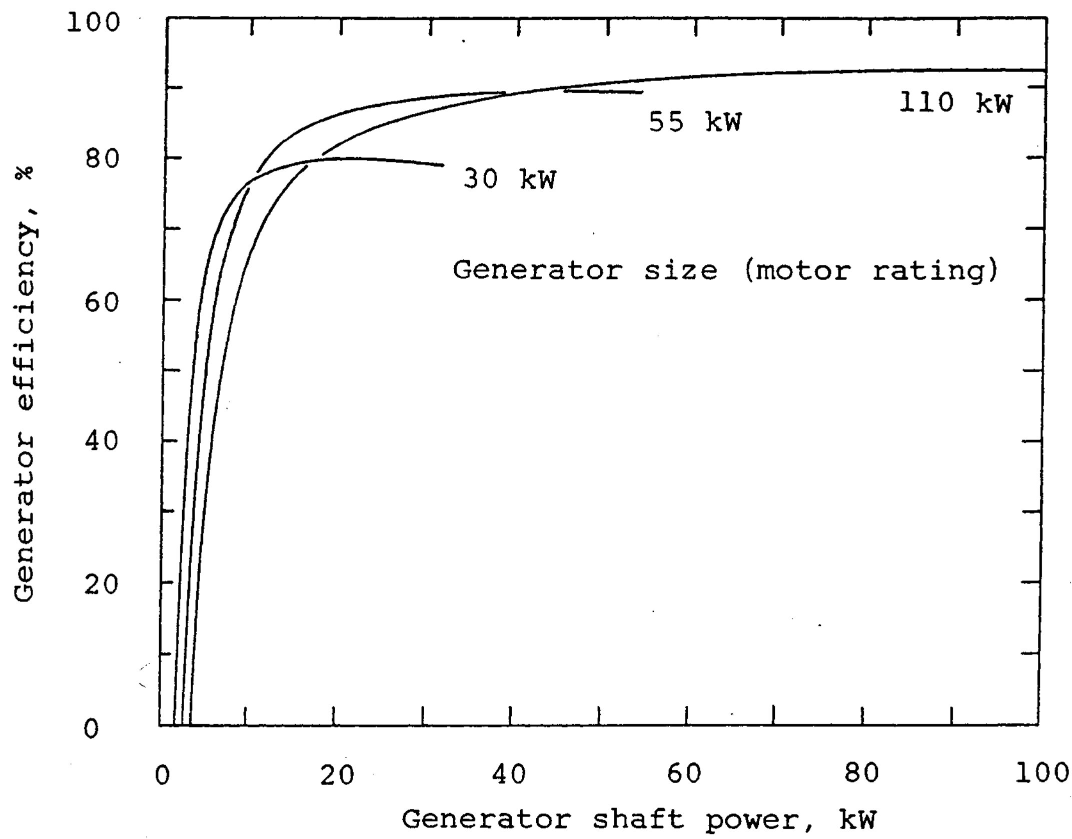

Figure 10.

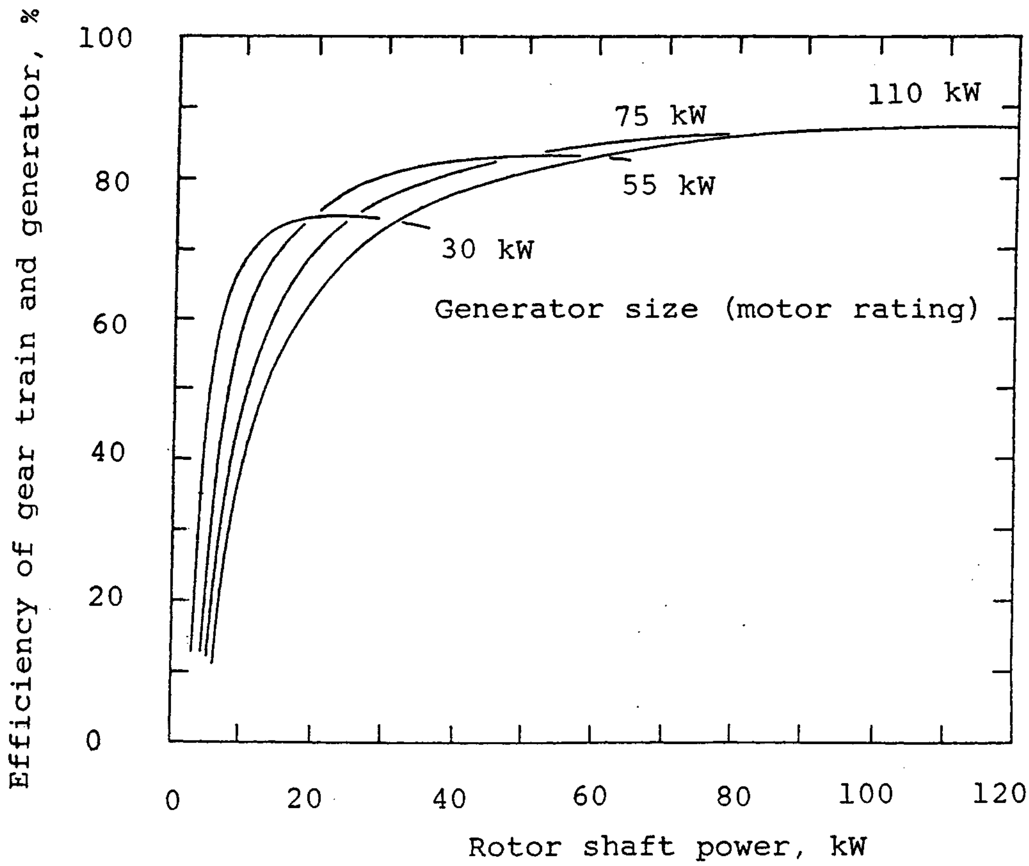

Figure 11.

Figures 10, 11 and 12 are sketches of the curves of efficiency used to convert the rotor power curves of Fig. 4 to electrical output power curves shown in Fig. 9. Figure 10 shows the generator efficiency for some generator sizes. In Fig. 11 the losses in the gear train is added to obtain the total efficiency. Figure 12 lis just another way to show the efficiency curves.

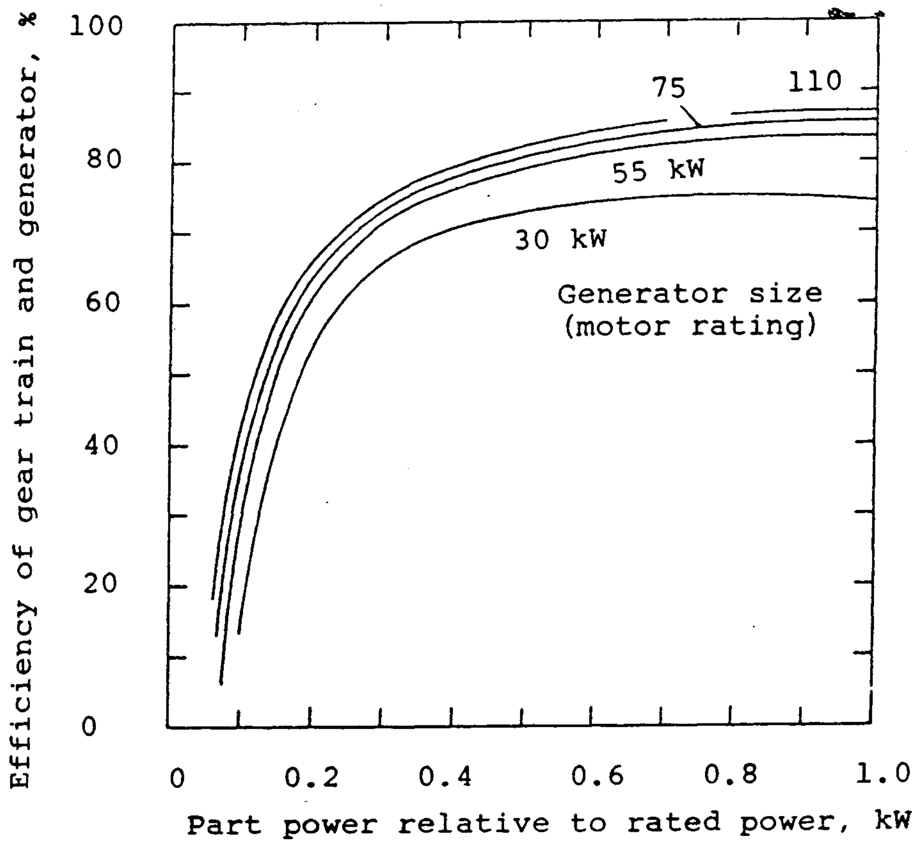

Figure 12.

In order to make the investigation realistic some actual sites with very different mean wind speeds for the imaginary erection of the wind turbines were selected. They are listed in Tables I to V; the sites in Tables II, III and IV are in California. In all tables the wind climates are described by the Weilbull-parameters, the scaling factor, C, and the shape factor (or form factor), K. It should be mentioned that the two parameters in Europe are denoted A and C, respectively.

Table I. Denmark

Reduction factor for elevation, $f_1 = 1.0$
Availability factor, $f_2 = 0.95$

	Mean wind $\overline{V}$, m/sec	Scale factor C, m/sec	Shape factor K
Landscape class 0-1	6.7	7.5	1.9
Landscape class 1-2	5.8	6.5	1.85

Table II. San Gorgonio Pass, Devers

Reduction factor for elevation, $f_1 = 0.96$
Availability factor, $f_2 = 0.95$

	Mean wind $\overline{V}$, m/sec	Scale factor C, m/sec	Shape factor K
Spring	9.6	11.5	1.5
Summer	9.1	9.7	1.6
Fall	5.8	7.3	1.6
Winter	5.3	6.2	1.5
Mean	7.5	-	-

Table III. San Gorgonio Pass, Site 7

Reduction factor for elevation, $f_1 = 0.96$
Availability factor, $f_2 = 0.95$

	Mean wind $\overline{V}$, m/sec	Scale factor C, m/sec	Shape factor K
Spring	9.6	11.8	3.0
Summer	9.1	10.5	2.9
Fall	4.3	5.4	1.9
Winter	5.2	6.8	2.5
Mean	7.1	-	-

Table IV. Solano County, Site S-04

Reduction factor for elevation, $f_1 = 0.98$
Availability factor, $f_2 = 0.95$

	Mean wind $\overline{V}$, m/sec	Scale factor C, m/sec	Shape factor K
Spring and summer	11.5	13.0	2.7
Fall and winter	5.7	5.7	1.1
Mean	8.6	-	-

Table V. Hawaii, Big Island, Kohala Mountain

Reduction factor for elevation, $f_1 = 0.9$
Availability factor, $f_2 = 0.95$

	Mean wind $\overline{V}$, m/sec	Scale factor C, m/sec	Shape factor K
Mean	11.9	13.4	2.2

The calculated annual energy production in MWh per year is shown in Fig. 9. The abscissa is the generator size, varying from 26.5 to 64 kW as shown in Fig. 8.

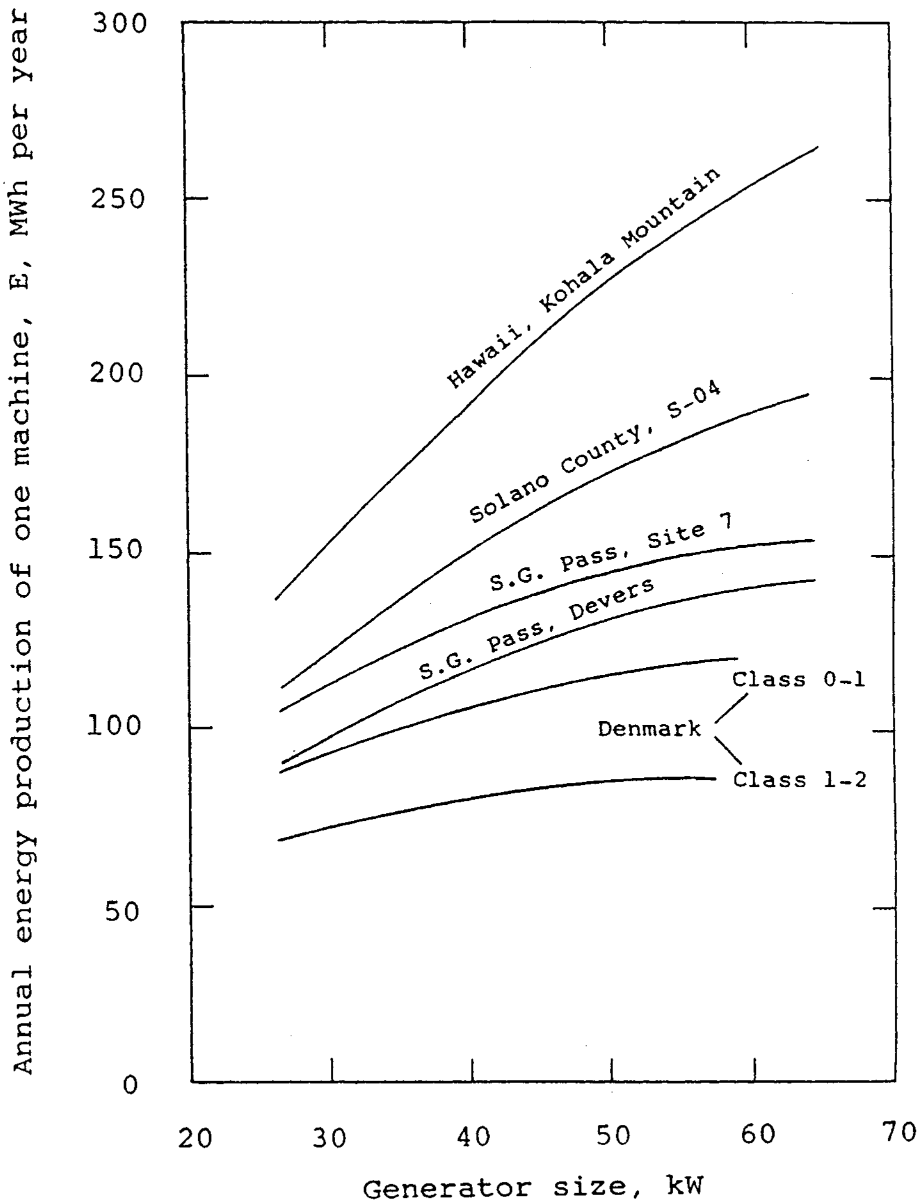

Figure 9.

Although such calculations are rather inaccurate the graph clearly shows the importance of making the right choice of generator size and rotational speed of the rotor. However, it must be borne in mind that other factors will influence this choice, such as the strength of the blades, possible other blade geometry or other incidence setting.

7. CONCLUSION.

The laws of similarity are shown to be very valuable in designing a wind turbine when modifications are needed to adapt the machine to a specific wind climate.

Risø - M - 2432

Risø National Laboratory

Risø - M - 2432

Title and author(s)

SIMPLIFIED LAWS OF SIMILARITY FOR WIND TURBINE ROTORS

Helge Petersen

Date May 1984

Department or group

The Test Station for Windmills

Group's own registration number(s)

24 pages + 5 tables + 13 illustrations

Copies to: Standard distribution

Abstract

The laws of similarity for the characteristics of a wind turbine rotor are of importance to the designer even during the initial design phase of a new wind turbine concept, but still more so when the designer wants to make adjustments or modifications to a wind turbine, in order to improve the machine, to optimize or adopt it to another wind climate than originally anticipated, or eventually redesign it by scaling up or down the rotor size. In the paper some examples are shown to illustrate the principles, and equations for the laws of similarity are derived under simplified conditions such as neglecting the influence of the variation of the Reynolds Number. Some comments are presented on comparison of stall- and pitch-regulated wind turbines and on two speed operation. The optimization of a stall-regulated wind turbine to different wind climates by variation of the generator size is dealt with.

Available on request from Risø Library, Risø National Laboratory (Risø Bibliotek , Forsøgsanlæg Risø), DK-4000 Roskilde, Denmark
Telephone: (03) 37 12 12, ext. 2262. Telex: 43116